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An object-wise approach to Categorical Algebra

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Abstract

How do we distinguish "good" objects in a setting with weak algebraic properties ... without using elements?

Many topics in mathematics involve the study of "objects" X endowed with certain properties, and "arrows" $f\colon X \to Y$ which preserve them. For example, homomorphisms between groups, linear maps between vector spaces, continuous maps between topological spaces. With that data we can consider new constructions or properties that can be, conveniently, represented by commutative diagrams of arrows. Diagrams are the essence of Category Theory.

By handling diagrams, Category Theory provides an element-free approach to many element-based mathematical fields, such as general algebraic ones. An element-free approach has the advantage of giving a better perception of whatever properties are being studied, as well as their possible generalisation to other settings. Indeed, an important boost towards the success and acceptance of Category Theory in the early 1950's was the introduction of the notion of *abelian category* [1], which defines an axiomatic categorical context (no elements involved) reflecting the main properties of abelian groups and modules.

In Algebra most contexts are non-abelian, such as the categories of groups, Lie algebras or rings. However, there may exist *abelian objects* (see [2]) amongst the objects of a non-abelian category which are worth emphasising. The notion of abelian object in Categorical Algebra has played a key role over the recent years, partly giving rise to abstract Commutator Theory (which gives a way to measure non-abelianness). In the category of groups (or monoids), the abelian objects are the abelian groups; in the category of Lie algebras over a field K, the abelian objects are K-vector spaces; in the category of rings, the abelian objects are the degenerate rings (those with zero multiplication).

Distinguishing "good" objects in weaker settings, such as the abelian case described above, is a common practice. This led us to develop in [3] an object-wise approach to some important notions occurring in Categorical Algebra. The main aim of that work was to provide

a categorical-algebraic characterisation of groups amongst monoids and of rings amongst semirings; of course, no elements involved.

The goal of this talk is to give an idea of how we may use a categorical approach to answer the initial question.

Keywords abelian category, abelian object, non-abelian Categorical Algebra

References

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